The Relationship Between Number of Distinct Categories and % Study Variation

The output for Gage R & R studies includes the number of distinct categories (ndc) and the percentage study variation (%Study Var, or %SV). To clearly understand how these two quantities are related, first consider how they are mathematically defined:

$$ndc = \sqrt{2}\frac{\sigma_{part}}{\sigma_{gage}} \qquad \%SV = \frac{\sigma_{gage}}{\sigma_{total}} \times 100$$

where σ represents the square root of the variance components. Using substitution, we can express the relationship between ndc and %SV as

$$\begin{aligned} ndc &= \sqrt{2}\frac{\sigma_{part}}{\sigma_{gage}} \\ ndc &= \sqrt{2}\frac{\sigma_{part}}{\sigma_{gage}}\left(\frac{\sigma_{gage}}{\sigma_{total}} \times 100\right) \cdot \left(\frac{1}{\%SV}\right) \\ ndc &= \sqrt{2}\frac{100 \cdot \sigma_{part}}{\sigma_{total}}\left(\frac{1}{\%SV}\right) \end{aligned}$$

The last equation shows that ndc and %SV are inversely proportional: the larger %SV is, the smaller the ndc is, and vice-versa. However, it also suggests that the value of ndc depends not only on %SV, but on the variance components as well.

To simplify the equation and represent ndc solely as a function of %SV, we can express the variance components in another way. The total variance is the sum of two variance components, one corresponding to gage repeatability and reproducibility and the other to part-to-part variation: $\sigma^2_{total} = \sigma^2_{gage} + \sigma^2_{part}$.

Solving for σ^2_{part} and dividing each side of the equation by σ^2_{total} yields

$$\frac{\sigma^2_{part}}{\sigma^2_{total}} = 1 - \frac{\sigma^2_{gage}}{\sigma^2_{total}}$$

Because $\frac{\%SV}{100} = \frac{\sigma_{gage}}{\sigma_{total}}$, the equation above can be rewritten as

$$\frac{\sigma_{part}}{\sigma_{total}} = \sqrt{1 - \frac{\sigma^2_{gage}}{\sigma^2_{total}}} = \sqrt{1 - \left(\frac{\%SV}{100}\right)^2}$$

Substituting this value into the previous equation for ndc gives the following simplified formula:

$$ndc = \frac{\sqrt{2}(100)}{\%SV}\sqrt{1-\left(\frac{\%SV}{100}\right)^2}$$

$$ndc = \frac{\sqrt{2}}{\%SV}\sqrt{10000-(\%SV)^2}$$

This equation clearly shows the relationship between ndc and %SV and can be used to calculate the number of distinct categories for a given percentage study variation. As shown in Table 1 below, the calculated ndc value is then truncated to obtain a whole number (integer). For example, if the calculated value is 15.8, mathematically you are not quite capable of differentiating between 16 categories. Therefore, Minitab is conservative and truncates the number of distinct categories to 15. For practical purposes, you can also round the calculated ndc values to obtain the number of distinct categories.

% Study Var	ndc
1	141.414 ~ 141
5	28.249 ~ 28
10	14.017 ~ 14
20	6.928 ~ 6
27	5.043 ~ 5
28	4.849 ~ 4
30	4.497 ~ 4
31	4.337 ~ 4
32	4.187 ~ 4
33	4.045 ~ 4
34	3.912 ~ 3

Table 1: Equivalences for NDC and %StudyVar

You can evaluate a measurement system by looking only at the number of distinct categories and using the following guidelines (based on the truncation method used by Minitab):

- **≥ 14 distinct categories** – the measurement system is acceptable.
- **4-13 distinct categories** – the measurement system is marginally acceptable, depending on the importance of the application, cost of measurement device, cost of repair, and other factors.
- **≤ 3 distinct categories** – the measurement system is unacceptable and should be improved.

These guidelines have some limitations. For example, in some cases when the % study variation is over 30% the number of distinct categories is 4. Therefore, a measurement system with 32% Study Var, which is unacceptable under the AIAG criteria for %Study Var, is acceptable under the ndc criteria. To avoid this discrepancy, some authors suggest only accepting a measurement system when it can distinguish between 5 or more categories. Although this fixes the original problem, it makes measurement systems with a 28-30% study variation unacceptable, because their corresponding ndc value equals 4.

To resolve this issue you can establish more specific guidelines based on the exact calculated ndc values, without truncating or rounding. For example, you could define an unacceptable measurement system based on an ndc < 4.497.